

LUMINOSITY FORMULAE

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LUMINOSITY CALCULATIONS

~~INTRODUCTION~~

A. Both Colliding Beams are bunched

$$L = N_1 N_2 f_{\text{encounter}} F$$

$$\begin{aligned}
 F = & \frac{2}{(2\pi)^{3/2} (\sigma_{\ell_1^2} + \sigma_{\ell_2^2})^{1/2}} \int_{-\infty}^{+\infty} \frac{ds}{(\sigma_{x_1^2} + \sigma_{x_2^2})^{1/2} (\sigma_{z_1^2} + \sigma_{z_2^2})^{1/2}} \times \\
 & \times \exp \left\{ -2s^2 \left(\frac{1}{\sigma_{\ell_1^2} + \sigma_{\ell_2^2}} + \frac{\alpha^2/4}{\sigma_{x_1^2} + \sigma_{x_2^2}} \right) \right\} \quad (1) \\
 = & \int_{-\infty}^{+\infty} G(s) ds
 \end{aligned}$$

$$\sigma^2 = \frac{\epsilon \beta(s)}{6\pi} \quad H \text{ and } V, \text{ zero dispersion}$$

ϵ emittance for 95% of beam with bi-gaussian distribution

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

N_1, N_2 , number of particles per bunch

σ_e , rms bunch length

α , total crossing angle assumed in the x -plane.

If both beams are made of B bunches each

$$f_{\text{encounter}} = B \cdot f_{\text{revolution}}$$

(2)

B. One Bunched Beam colliding with an Unbunched Beam

$$L = N_u N_B f_{\text{encounter}} F$$

$$F = \frac{1}{2\pi^2 R} \int_{-\infty}^{+\infty} \frac{ds}{(\sigma_{x_0}^2 + \sigma_{x_B}^2)^{1/2} (\sigma_{z_0}^2 + \sigma_{z_B}^2)^{1/2}} \times \\ \times \exp \left\{ - \frac{\alpha^2 s^2}{2(\sigma_{x_0}^2 + \sigma_{x_B}^2)} \right\} \quad (2)$$

$$= \int_{-\infty}^{+\infty} G(s) ds$$

$2\pi R$, circumference of reference orbit ~~assumed~~ to be assumed to be the same for both beams
 N_u , total number of particles in the unbunched beam
 N_B , no. of particles per bunch in the bunched beam
 B , no. of bunches in the bunched beam

$f_{\text{encounter}} = B$ revolution

Extreme Cases

A. Both Colliding Beams are bunched

Long Bunches : $\sigma_e \gg 2\sigma_x/\alpha$

σ_x and σ_z are constant ~~over~~ over interaction region

Define effective cross-sections and length

$$\overline{\sigma_x}^2 = \frac{\sigma_{x1}^2 + \sigma_{x2}^2}{2}$$

$$\overline{\sigma_z}^2 = \frac{\sigma_{z1}^2 + \sigma_{z2}^2}{2}$$

$$\overline{\sigma_e}^2 = \frac{\sigma_{e1}^2 + \sigma_{e2}^2}{2}$$

We have

$$L = \frac{N_1 N_2 B \text{ revolution}}{2\pi \alpha \overline{\sigma_e} \overline{\sigma_z}} \quad (3)$$

having assumed that both beams have the same number of bunches B .

(4)

B. One Bunched Beam colliding with an Unbunched Beam

σ_x and σ_z are constant over interaction region

Define effective cross-sections

$$\bar{\sigma}_x^2 = \frac{\sigma_{xv}^2 + \sigma_{xB}^2}{2}$$

$$\bar{\sigma}_z^2 = \frac{\sigma_{zv}^2 + \sigma_{zB}^2}{2}$$

We have

$$L = \frac{N_v N_B \text{ revolution}}{2\pi \alpha (\sqrt{\pi} R) \bar{\sigma}_z^2} \quad (4)$$

Comment

For crossing at a large angle and for long bunches or co-axial beams the equations shown above are correct as long

$$\sqrt{2} \frac{\sigma_x}{d} \ll l \quad (5)$$

where l is half of the length of the vacuum chamber shared by both beams unshielded from each other.

Eq. (3) and (4) are derived assuming eq. (5) holds.

If relation (5) does not hold the integration limits in eq. (1) and (2) are to be replaced by $-l$ and $+l$ respectively instead of $-\infty$ and $+\infty$.

C. Short Beamlets Colliding Head-on ~~(6.6)~~

Extreme Case from eq. (1) with $\alpha=0$ -

$\bar{\sigma}_x$ and $\bar{\sigma}_z$ are constant ~~of~~ over interaction region -

Assume $\bar{\sigma}_e \ll l$

Then we have with the usual notation

$$L = \frac{N_1 N_2 B \text{ fravolution}}{4\pi \bar{\sigma}_x \bar{\sigma}_z}$$

having again assumed that both beamlets have the same number of beamlets B -

References

Lloyd Smith
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, FN - 271 , December 1974